Time correlation of momentum in a collisionless plasma due to large fluctuations

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The present paper shows that in one-dimensional collisionless plasma, due to large fluctuation, the time correlation of momentum asymptotically behaves inverse quadratically with time. The long-time tail correlation is closely related to nonlinear Landau damping [M.B. Isichenko, Phys. Rev. Lett. **78**, 2369 (1997)]. Furthermore, it reveals the possible relationship between fluctuation and dispersion. The result indicates that the long-time tail correlation can also occur in a reversibly dispersive system, where the previous theories are invalid. A by-product proves that it may be efficient to investigate the dispersive features of collisionless plasma in view of time correlation. [S1063-651X(99)03206-7]

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I. INTRODUCTION

Extensive attention has been paid to long-time tail correlation since Alder and Wainwright inspected the behavior in molecular dynamical simulation in 1967 [1]. Nowadays it is well known that such an asymptotic behavior is related to the long wavelength dissipative hydrodynamics [2]. Indeed, detailed calculations show that the time correlation involves hydrodynamic shear and thermal modes. However, many questions remain unanswered. For instance, the question of whether long-time tail correlation exists beyond irreversibly dissipative system is of great interest to us.

In the present work, based on the celebrated Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) theory and the mean field approximation, the time correlation of momentum of one-dimensional (1D) collisionless plasma is derived. Furthermore, the time correlation is proved to behave inverse quadratically with time asymptotically, due to large fluctuation.

II. THE DERIVATION OF THE TIME CORRELATION OF MOMENTUM

Consider an N-body interacting system and introduce the so-called two-ensemble distribution function, say, $f(1',2',\ldots,N',1,2,\ldots,N;t+\tau,t)$, which represents the number density of such states as in the neighborhood of the state $(1,2,\ldots,N)$ at time t while in the neighborhood of the state $(1',2',\ldots,N')$ at time $(t+\tau)$. Here $k=(\mathbf{x}_k,\mathbf{p}_k)$, $k'=(\mathbf{x}'_k,\mathbf{p}'_k)$, and $\mathbf{x}_k s,\mathbf{p}_k s(\mathbf{x}'_k s,\mathbf{p}'_k s)$, are the coordinates and momenta of particles. Then, the well-known continuity equation and the measure preserving yield reads

$$\left(\frac{\partial}{\partial\tau}+\sum_{k=1}^{N}\mathbf{v}'_k\cdot\boldsymbol{\nabla}_{\mathbf{x}'_k}+\sum_{k=1}^{N}\mathbf{F}'_k\cdot\nabla_{p'_k}\right)\times f(1',\ldots,N',1,\ldots,N;t+\tau,t)=0, \tag{1}$$

where $\mathbf{F}'_k$ is the force exerted on particle k at time $(t+\tau)$. Moreover, it is not difficult to obtain the corresponding initial condition

$$f(1',\ldots,N',1,\ldots,N;t,t)=f(1,\ldots,N;t)\,\delta(1'-1)\ldots\delta(N'-N), \tag{2}$$

where $f(1,2,\ldots,N;t)$ is the N-body distribution function. Equation (1) is similar to the usual Liouville equation, but not so trivial. It can be seen that Eqs. (1) and (2) are greatly advantageous to systematical introduction of microscopic approximation into the discussion of fluctuation in the absence of the complete knowledge of the N-body problem. In addition, emphasis is placed on that the initial condition is not arbitrary.

For the one-body dynamical variable (e.g., momentum $\mathbf{p}$), the local time correlation is

$$\langle\mathbf{p}(t)\mathbf{p}(t+\tau)\rangle_x=\int\sum_{j=1}^{N}\sum_{k=1}^{N}\mathbf{p}_j\,\delta(\mathbf{x}-\mathbf{x}_j)\mathbf{p}'_k\,\delta(\mathbf{x}-\mathbf{x}'_k)\times f(1',\ldots,N',1,\ldots,N;t+\tau,t)\times d1'\cdots dN'\,d1\cdots dN. \tag{3}$$

With the employment of the symmetry of arguments and the definitions as follows:

$$f_1(1',1;t+\tau,t)=N\int f(1',\ldots,N',1,\ldots,N;t+\tau,t)\times d2'\cdots dN'\,d2\cdots dN, \tag{4}$$

$$f_1(1',2;t+\tau,t)=N(N-1)\int f(1',\ldots,N',1,\ldots,N;t+\tau,t)\times d2'\cdots dN'\,d1d3\cdots dN, \tag{5}$$

Eq. (3) is reduced to

$$\langle \mathbf{p}(t)\mathbf{p}(t+\tau)\rangle_x=\int \mathbf{p}_1\mathbf{p}_1'\delta(\mathbf{x}-\mathbf{x}_1)\delta(\mathbf{x}-\mathbf{x}_1')f_1(1',1;t+\tau,t)\times d1'd1+\int \mathbf{p}_2\mathbf{p}_1'\delta(\mathbf{x}-\mathbf{x}_2)\delta(\mathbf{x}-\mathbf{x}_1')\times f_1(1',2;t+\tau,t)d1'd2. \quad (6)$$

So far, all the results are exact, but formal. In the rest of this paper, they are put into practice in the case of 1D collisionless plasma (hence the bold type of $\mathbf{x}_k$'s and $\mathbf{p}_k$'s is not in use from now on). Based on the BBGKY theory and the mean field approximation, the following equations can be obtained from Eqs. (1), (4), and (5):

$$\left[\frac{\partial}{\partial\tau}+v_1'\frac{\partial}{\partial x_1'}-eE'(x_1',\tau)\frac{\partial}{\partial p_1'}\right]f_1(1',1;t+\tau,t)=0, \quad (7)$$

$$\frac{\partial}{\partial x_1'}E'(x_1',\tau)=-4\pi e\left[\int f_1(1',1;t+\tau,t)dp_1'-Zn_+\right], \quad (8)$$

and

$$\left[\frac{\partial}{\partial\tau}+v_1'\frac{\partial}{\partial x_1'}-eE'(x_1',\tau)\frac{\partial}{\partial p_1'}\right]f_1(1',2;t+\tau,t)=0, \quad (9)$$

$$\frac{\partial}{\partial x_1'}E'(x_1',\tau)=-4\pi e\left[\int f_1(1',2;t+\tau,t)dp_1'-Zn_+\right], \quad (10)$$

in the absence of external field. In Eqs. (8) and (10), the uniform ion background has been taken into account, Z is the ion electric charge number, and n_+ is the ion number density. In fact, Eqs. (7)–(10) can be derived on the basis of more rigorous mathematics. The crucial point is that if the parameters $1,\ldots,N$ in Eq. (1) are neglected for the moment, Eq. (1) turns into the ordinary Liouville equation, thus the perturbation theory developed for the treatment of the Liouville equation [3] can be applied. Therefore, expanding Eq. (1) up to the first order of the interaction constant e will retrieve the result. Correspondingly, the combination of Eqs. (2), (4), and (5) yields the initial condition

$$f_1(1',1;t,t)=N\int f(1,\ldots,N;t)\delta(1'-1)d2\cdots dN=f_1(1;t)\delta(1'-1), \quad (11)$$

$$f_1(1',2;t,t)=N(N-1)\int f(1,\ldots,N;t)\delta(1'-1)\times d1d3\cdots dN=f_2(1',2;t). \quad (12)$$

Here $f_1(1;t)$ and $f_2(1',2;t)$ are the one-body and two-body distribution function, respectively.

Equation (7) or (9) is nothing but the one-body Liouville equation except that the external electric field is mean field. It is easy to obtain the formal solutions from Eqs. (7), (9), (11), and (12):

$$\begin{aligned}f_1(1',1;t+\tau,t)&=f_1(1;t)\delta(x_1'^*-x_1)\delta(p_1'^*-p_1)\\&=f_1(1;t)\delta\left(x_1'-\int_0^\tau v_1'(x_1'^*,p_1'^*,s)ds-x_1\right)\\&\quad\times\delta\left(p_1'+\int_0^\tau eE'[x_1'(x_1'^*,p_1'^*,s),s]ds-p_1\right),\end{aligned} \quad (13)$$

$$\begin{aligned}f_1(1',2;t+\tau,t)&=f_2(x_1'^*,p_1'^*,x_2,p_2;t)\\&=f_2\left(x_1'-\int_0^\tau v_1'(x_1'^*,p_1'^*,s)ds,p_1'\right.\\&\quad\left.+\int_0^\tau eE'[x_1'(x_1'^*,p_1'^*,s),s]ds,x_2,p_2;t\right).\end{aligned} \quad (14)$$

Here (and henceforth) we denote the initial values of x_1', v_1', and p_1' at $\tau=0$ by $x_1'^*$, $v_1'^*$, and $p_1'^*$. $x_1'(s)$, $v_1'(s)$ are the trajectory and velocity of the motion under the mean field with the initial conditions $x_1'(0)=x_1'^*$, $v_1'(0)=v_1'^*$. In the derivation of the first equalities of Eqs. (13) and (14), the measure preserving of the motion under the mean field is used. The property of measure preserving is not so apparent as in Hamiltonian dynamics. In fact, for the infinitesimal time interval $\delta\tau$,

$$x_1'=x_1'(\tau)+v_1'(\tau)\delta\tau,\quad p_1'=p_1'(\tau)-eE'[x_1'(\tau),\tau]\delta\tau, \quad (15)$$

and the Jacobian is

$$\begin{aligned}J(\tau+\delta\tau)&=\frac{\partial(x_1',p_1')}{\partial(x_1'^*,p_1'^*)}\\&=\frac{\partial[x_1'(\tau+\delta\tau),p_1'(\tau+\delta\tau)]}{\partial[x_1'(\tau),p_1'(\tau)]}\frac{\partial[x_1'(\tau),p_1'(\tau)]}{\partial(x_1'^*,p_1'^*)}\\&=\det\begin{pmatrix}1 & -\delta\tau\frac{\partial}{\partial x_1'}eE'[x_1'(\tau),\tau]\\ \delta\tau/m & 1\end{pmatrix}J(\tau)\\&=J(\tau)[1+o(\tau)].\end{aligned} \quad (16)$$

Hence $(d/d\tau)J(\tau)=0$ and $J(\tau)=1$, which implies $dx_1'dp_1'=dx_1'^*dp_1'^*$. The point is the E' is not explicitly dependent on p_1', otherwise no conclusive result could be brought about. If Eqs. (6) and (13) combined, we find that the first term on the right-hand side of Eq. (6) vanishes. The proof is straightforward,

$$\begin{aligned}\alpha&=\int p_1p_1'\delta(x-x_1)\delta(x-x_1')f_1(1',1;t+\tau,t)d1'd1\\&=\int p_1p_1'\delta(x-x_1)\delta(x-x_1')f_1(1;t)\\&\quad\times\delta\left(x_1'-\int_0^\tau v_1'(x_1'^*,p_1'^*,s)ds-x_1\right)\end{aligned}$$

$$\times\delta\left(p_1'+\int_0^\tau eE'[x_1'(x_1'^*,p_1'^*,s),s]ds-p_1\right)d1'd1$$

$$=\int p_1p_1'f_1(x,p;t)\delta\left(\int_0^\tau v_1'(x_1'^*,p_1'^*,s)ds\right)$$
$$\times\delta\left(p_1'+\int_0^\tau eE'[x_1'(x_1'^*,p_1'^*,s),s]ds-p_1\right)$$
$$\times dp_1'dp_1, \tag{17}$$

in general, particularly for $\tau\to\infty$, $\int_0^\tau v_1'(x_1'^*,p_1'^*,s)ds$ does not vanish and α equals zero. Therefore, a meaningful result is found that the correlation of the same particle at different time does not contribute to the time correlation of the momentum at all. Then, employing the measure preserving and substituting the first equality of Eq. (14) into Eq. (6), one may get

$$\langle p(t)p(\tau+t)\rangle_x=\int p_2p_1'\delta(x-x_2)\delta(x-x_1')$$
$$\times f_2(x_1'^*,p_1'^*,x_2,p_2;t)dx_1'^*dp_1'^*$$
$$\times dx_2dp_2\quad\text{for}\quad\tau\to\infty. \tag{18}$$

Since in most cases, only total current-current correlation is of interest, just take average on $\langle p(t)p(\tau+t)\rangle_x$ over the whole space. Employing Eq. (18), the spatial average of $\langle p(t)p(\tau+t)\rangle_x$ is

$$\langle p(t)p(\tau+t)\rangle_{\rm ave}$$
$$=\lim_{L\to\infty}L^{-1}\int_{-L/2}^{L/2}dx\langle p(t)p(\tau+t)\rangle_x$$
$$=\lim_{L\to\infty}L^{-1}\int p_2p_1'\delta(x_1'-x_2)f_2(x_1'^*,p_1'^*,x_2,p_2;t)$$
$$\times dx_1'^*dp_1'^*dx_2dp_2$$
$$=\lim_{L\to\infty}L^{-1}\int p_2\left[p_1'^*-\int_0^\tau eE'[x_1'(x_1'^*,p_1'^*,s),s]ds\right]$$
$$\times\delta\left[x_1'^*+\int_0^\tau v_1'(x_1'^*,p_1'^*,s)ds-x_2\right]$$
$$\times f_2(x_1'^*,p_1'^*,x_2,p_2;t)dx_1'^*dp_1'^*dx_2dp_2$$
$$=\lim_{L\to\infty}L^{-1}\int p_2\left[p_1'^*-\int_0^\tau eE'[x_1'(x_1'^*,p_1'^*,s),s]ds\right]$$
$$\times f_2\left(x_1'^*,p_1'^*,x_1'^*+\int_0^\tau v_1'(x_1'^*,p_1'^*,s)ds,p_2;t\right)$$
$$\times dx_1'^*dp_1'^*dp_2, \tag{19}$$

where L is the 1D volume of the system. f_2 can be written as

$$f_2(x_1,p_1,x_2,p_2;t)=f_1(x_1,p_1;t)f_1(x_2,p_2;t)$$
$$+g(x_1,p_1,x_2,p_2;t)$$
$$\approx f_1(x_1,p_1;t)f_1(x_1,p_2;t)$$
$$+\left[(x_2-x_1)\frac{\partial}{\partial x_1}f_1(x_1,p_2;t)\right]$$
$$\times f_1(x_1,p_1;t)+g(x_1,p_1,x_2,p_2;t). \tag{20}$$

In the last line, the spatial inhomogeneity accounts for the second term and g is the pair correlation. The second term is the order of $|\int_0^\tau v_1'ds|/L_{\rm inh}$, where $L_{\rm inh}$ is the characteristic scale of spatial inhomogeneity. If the plasma is weakly inhomogeneous, $L_{\rm inh}$ is sufficiently large, $|\int_0^\tau v_1'ds|/L_{\rm inh}\ll 1$ so this term can be neglected. In addition, g is neglected when use of the mean field approximation is made. In this way, $f_2(x_1'^*,p_1'^*,x_1'^*+\int_0^\tau v_1'(x_1'^*,p_1'^*,s)ds,p_2;t)$ is simplified as $f_1(x_1'^*,p_1'^*;t)f_1(x_1'^*,p_2;t)$ so that Eq. (19) becomes

$$\langle p(t)p(\tau+t)\rangle_{\rm ave}$$
$$=\lim_{L\to\infty}L^{-1}\int p_2\left[p_1'^*-\int_0^\tau eE'[x_1'(x_1'^*,p_1'^*,s),s]ds\right]$$
$$\times f_1(x_1'^*,p_1'^*;t)f_1(x_1'^*,p_2';t)dx_1'^*dp_1'^*dp_2$$
$$=\lim_{L\to\infty}L^{-1}\int p_2p_1'^*f_1(x_1'^*,p_1'^*;t)f_1(x_1'^*,p_2;t)$$
$$\times dx_1'^*dp_1'^*dp_2$$
$$-\lim_{L\to\infty}L^{-1}\int dx_1'^*dp_1'^*dp_2f_1(x_1'^*,p_1'^*;t)$$
$$\times f_1(x_1'^*,p_2;t)p_2\int_0^\tau eE'[x_1'(x_1'^*,p_1'^*,s),s]ds. \tag{21}$$

To be specific, only the τ-dependent part, i.e., the second term, is of interest

$$\langle p(t)p(\tau+t)\rangle_{\rm ave}\sim\lim_{L\to\infty}L^{-1}\int dx_1'^*dp_1'^*dp_2f_1(x_1'^*,p_1'^*;t)$$
$$\times f_1(x_1'^*,p_2;t)p_2\int_0^\tau eE'$$
$$\times[x_1'(x_1'^*,p_1'^*,s),s]ds\quad\text{for}\quad\tau\to\infty. \tag{22}$$

III. AN ANALYSIS OF THE LONG-TIME TAILS CORRELATION OF MOMENTUM

One can see from Eqs. (10) and (18), it is E', to which the correlation of two different particles at different time gives rise, that contributes to the time correlation of momentum. Strictly speaking, in view of weak inhomogeneity, E' is greatly attributed to the pair correlation g, which is part of f_2 [see Eq. (20)]. If the fluctuation, hence E', is small, the solution to Eqs. (9) and (10) can be found according to the famous linear Landau damping theory [4]. Therefore, it is natural to expect that the time correlation decays exponentially. However, the method does not hold for large fluctuation and a new version must be proposed. Fortunately, the method developed in Ref. [5] can be applied because of the analogy between Eqs. (9) and (10) and the ordinary Vlasov-Poisson equations.

First, impose periodic boundary condition on $f_1(1',2,t+\tau,t)$ in x_1' and the period is l, which is sufficiently large. Expanding $f_1(1',2,t+\tau,t)$ and $E'(x_1',t+\tau)$ in a Fourier series

$$f_1(1',2;t+\tau,t)=(2\pi l)^{-1}\sum_k f_1(k,p_1',x_2,p_2;t+\tau,t)\times\exp(ikx_1'), \tag{23}$$

$$E'(x_1',\tau)=(2\pi l)^{-1}\sum_k E_k'(\tau)\exp(ikx_1'). \tag{24}$$

Differentiating Eq. (10) with respect to x_1' and taking Eqs. (23) and (24) into account, we then obtain

$$E_k'(\tau)=-8\pi^2 e(ik)^{-1}\int dx_1' dp_1' f_1(1',2;t+\tau,t)\times\exp(-ikx_1'). \tag{25}$$

By making use of the Liouville theorem, this can be transformed into

$$\begin{aligned}E_k'(\tau)&=-8\pi^2 e(ik)^{-1}\int dx_1''^{*}dp_1''^{*} f_1(x_1''^{*},p_1''^{*},x_2,p_2;t,t)\\&\quad\times\exp[-ikx_1'(x_1''^{*},p_1''^{*},\tau)]\\&=-8\pi^2 e(ik)^{-1}\int dx_1''^{*}dp_1''^{*} f_1(x_1''^{*},p_1''^{*},x_2,p_2)\\&\quad\times\exp[-ikx_1'(x_1''^{*},p_1''^{*},\tau)].\end{aligned} \tag{26}$$

Thus

$$\int_0^\tau ds\, E_k'(s)=-8\pi^2 e(ik)^{-1}\int dx_1''^{*}dp_1''^{*}\times f_1(x_1''^{*},p_1''^{*},x_2,p_2)\int_0^\tau ds\times\exp[-ikx_1'(x_1''^{*},p_1''^{*},s)]. \tag{27}$$

For large τ, due to the decay of the fluctuation of the electric field, the particle tends to uniform motion with velocity u. So we can facilitate the above equation as

$$\int^\tau ds\, E_k'(s)\sim-8\pi^2 e(ik)^{-1}\int dx_1''^{*}dp_1''^{*}\times f_2(x_1''^{*},p_1''^{*},x_2,p_2)\int^\tau ds\times\exp[-iksu(x_1''^{*},p_1''^{*})]\quad\text{for}\quad\tau\to\infty. \tag{28}$$

Exerting inverse Fourier series expansion on Eq. (26) gives

$$\begin{aligned}&\int_0^\tau ds\, E'[x_1'(x_1'^{*},p_1'^{*},s),s]\\&=-4\pi e l^{-1}\sum_k (ik)^{-1}\int dx_1''^{*}dp_1''^{*}\\&\quad\times f_2(x_1''^{*},p_1''^{*},x_2,p_2)\int_0^\tau ds\,\exp[ikx_1'(x_1'^{*},p_1'^{*},s)]\\&\quad\times\exp[-ikx_1'(x_1''^{*},p_1''^{*},s)]\\&\sim-4\pi e l^{-1}\sum_k (ik)^{-1}\int dx_1''^{*}dp_1''^{*}\\&\quad\times f_2(x_1''^{*},p_1''^{*},x_2,p_2)\int^\tau ds\,\exp[iksu(x_1'^{*},p_1'^{*})]\\&\quad\times\exp[-iksu(x_1''^{*},p_1''^{*})]\\&=4\pi e l^{-1}\sum_k k^{-2}\int dx_1''^{*}dp_1''^{*} f_2(x_1''^{*},p_1''^{*},x_2,p_2)\\&\quad\times[u(x_1'^{*},p_1'^{*})-u(x_1''^{*},p_1''^{*})]^{-1}\\&\quad\times\exp[ik\tau u(x_1'^{*},p_1'^{*})]\exp[-ik\tau u(x_1''^{*},p_1''^{*})],\end{aligned} \tag{29}$$

for $\tau\to\infty$. Substituting this into Eq. (22) yields

$$\begin{aligned}\langle p(t)p(\tau+t)\rangle_{\text{ave}}&\sim\lim_{L\to\infty}L^{-1}\int_{-L/2}^{L/2}dx_1'^{*}\int dp_1'^{*}\int dp_2\\&\quad\times f_1(x_1'^{*},p_1'^{*};t)f_1(x_1'^{*},p_2;t)p_2\\&\quad\times 4\pi e l^{-1}\sum_k k^{-2}\exp[ik\tau u(x_1'^{*},p_1'^{*})]\\&\quad\times\int dx_1''^{*}dp_1''^{*} f_2(x_1''^{*},p_1''^{*},x_2,p_2)\\&\quad\times[u(x_1'^{*},p_1'^{*})-u(x_1''^{*},p_1''^{*})]^{-1}\\&\quad\times\exp[-iksu(x_1''^{*},p_1''^{*})]\quad\text{for }\tau\to\infty.\end{aligned} \tag{30}$$

It has been proven that there exist stationary points for $u(x_1'^{*},p_1'^{*})$ with the assistance of the computer simulation in Ref. [5]. Allowance for the rapid vibration of exp($ikru$) at large τ and the assumption of the smoothness of $f_2(x_1''^{*},p_1''^{*},x_2,p_2)[u(x_1'^{*},p_1'^{*})-u(x_1''^{*},p_1''^{*})]^{-1}$, the integration over $x_1''^{*},p_1''^{*}$ is dominated by the vicinity of the stationary point $u(x_1''^{*},p_1''^{*})$, hence is the order of τ^{-1}.

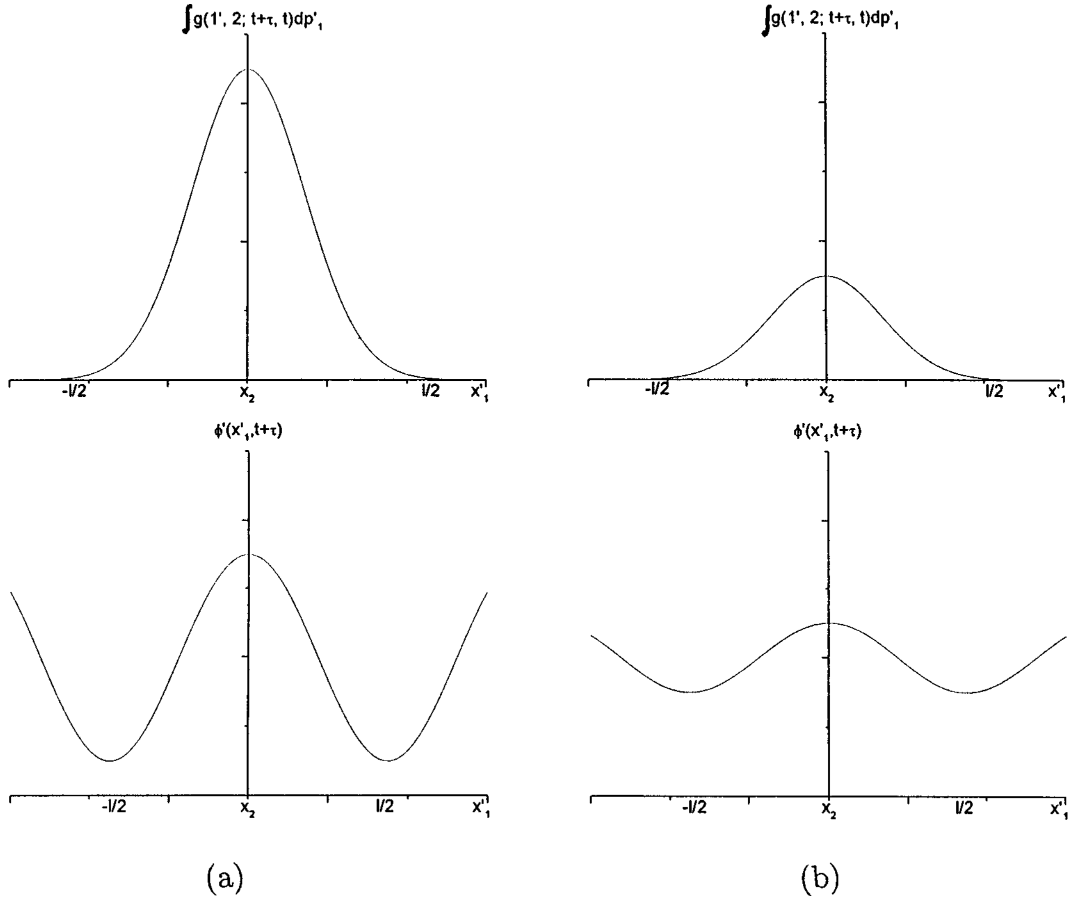

FIG. 1. The schematic illustration of the long-time tails. Top: The two-time correlation of two different particles. Only pair correlation is designated since E' is greatly attributed to g in view of weak inhomogeneity. Bottom: The fluctuating potential arising from the two-time correlation. (a) For the strong two-time correlation, the magnitude of potential is large. (b) For the two-time correlation which is weak or decays fast, the magnitude of potential is small. (It is essential that the integration over p_1' be performed just for the space-time correlation. The origin of the x_1' axis is x_2. Here x_2, p_2, and t should be understood as parameters.)

Moreover, it is reasonable to suppose that after the integration over $x_1''^*,p_1''^*$, the $x_1'^*,p_1'^*$-dependent integrand is also smooth. For the same reason, the integration over $x_1'^*,p_1'^*$ is the order of τ^{-1}, thus $\langle p(t)p(\tau+t)\rangle_{\text{ave}}$ is the order of τ^{-2}:

$$\langle p(t)p(\tau+t)\rangle_{\text{ave}}\sim\tau^{-2} \quad \text{for} \quad \tau\to\infty. \tag{31}$$

How the large fluctuation leads to the long-time tails can generally be illustrated by Fig. 1. In the sense of statistics, if particle 2 is localized at x_2 at time t, due to two-time correlation $f_1(1',2;t+\tau,t)$, there is electric charge distribution with respect to the position x_2 in the virtual space x_1' at time $(t+\tau)$, which gives rise to virtual potential $\phi'(x_1',t+\tau)$ according to Eq. (10). If either the magnitude of ϕ' is small or ϕ' decays sufficiently fast, no bouncing in the virtual potential occurs, and the theory would be linear, hence the time correlation decays exponentially. Otherwise, particle 1 can bounce in the potential in the course of time $(t+\tau)$ so that the time correlation decays algebraically (see Ref. [5]).

IV. CONCLUSIONS

In conclusion, it has been demonstrated that there exists long-time tails correlation as τ^{-2} in 1D collisionless plasma, arising from large fluctuation. The physical picture can be included in Fig. 1. Since $\int f_1(1',2;t+\tau,t)dp_1'$ is x_1' dependent, according to Eq. (10), the virtual potential $\phi'(x_1',t+\tau)$ occurs, which plays a central role in the appearance of the long-time tails. If the two-time correlation of two different particles is weak or decays fast, particle 1, in the course of time $(t+\tau)$ (t refers to a parameter), escapes from the potential, in turn, leading to exponential decay. Otherwise, particle 1 is readily bounded up in the virtual potential, which accounts for the occurrence of the long-time tails. Therein the close relationship between the time correlation (i.e., fluctuation) and dispersion, which in the given context refers to nonlinear Landau damping, can be thrashed out. It must be stressed that the mechanism of the long-time tail correlation in the context differs greatly from what has been interpreted by previous theories, i.e., the long wavelength dissipative hydrodynamics. In essence, the system considered here is reversibly dispersive. Moreover, with the aid of the traditional BBGKY theory, the direct analysis of the time correlation function has given the authors insight into the dispersive features of the collisionless plasma.

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[1] B. J. Alder and T. E. Wainwright, Phys. Rev. Lett. **18**, 998 (1967).
[2] J. R. Dorfmann and E. G. D. Cohen, Phys. Rev. Lett. **25**, 1257 (1970); M. H. Ernst, E. H. Hauge, and J. M. van Leeuwen, *ibid.* **25**, 1254 (1970); Phys. Rev. A **4**, 2055 (1971); Y. Pomeau, Phys. Lett. A **27**, 601 (1968); R. Kubo, M. Toda, and N. Hashitsume, *Statistical Physics II: Nonequilibrium Statistical Mechanics*, 2nd ed. (Springer-Verlag, Heidelberg, 1992).
[3] R. Balescu, *Equilibrium and Nonequilibrium Statistical Mechanics* (Wiley-Interscience, New York, 1975).
[4] L. D. Landau, J. Phys. (Moscow) **10**, 25 (1946).
[5] M. B. Isichenko, Phys. Rev. Lett. **78**, 2369 (1997).